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B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2012.

Fourth Semester

Common to ECE and Bio Medical Engineering

MA 2261 / 181403/ MA 45/ MA 1253 / 10177 PR 401 / 080380009 —
PROBABILITY AND RANDOM PROCESSES

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find C, if $P[X = n] = C\left(\frac{2}{3}\right)^n$; $n = 1,2,\ldots$

2. The probability that a man shooting a target is 1/4. How many times must he fire so that the probability of his hitting the target atleast once is more than 2/3?

3. Let X and Y be two discrete random variables with joint probability mass function

$$P(X = x, Y = y) = \begin{cases} \frac{1}{18}(2x + y), & x = 1,2 \text{ and } y = 1,2 \\ 0, & \text{otherwise.} \end{cases}$$

Find the marginal probability mass functions of X and Y.

4. State Central Limit Theorem for iid random variables.

5. Define wide-sense stationary random process.

6. If $\{X(t)\}$ is a normal process with $\mu(t) = 10$ and $C(t_1, t_2) = 16e^{-|t_1 - t_2|}$ find the variance of $X(10) - X(6)$.

7. The autocorrelation function of a stationary random process is $R(\tau) = 16 + \frac{9}{1 + 6\tau^2}$. Find the mean and variance of the process.

8. Prove that $S_{xy}(\omega) = S_{yx}(-\omega)$.

9. Prove that the system $y(t) = \int_{-\infty}^{\infty} h(u).X(t-u)du$ is a linear time-invariant system.

10. What is unit impulse response of a system? Why is it called so?

PART B — (5 × 16 = 80 marks)

11. (a) (i) A random variable X has the following probability distribution.

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

Find :

(1) The value of K

(2) $P(1.5 < X < 4.5 \mid X > 2)$ and

(3) The smallest value of n for which $P(X \leq n) > \frac{1}{2}$. (8)

(ii) Find the M.G.F. of the random variable X having the probability density function

$$f(x) = \begin{cases} \frac{x}{4} e^{\frac{-x}{2}}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Also deduce the first four moments about the origin. (8)

Or

(b) (i) Given that X is distributed normally, if $P[X < 45] = 0.31$ and $P[X > 64] = 0.08$, find the mean and standard deviation of the distribution. (8)

(ii) The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$

(1) What is the probability that the repair time exceeds 2 hours?

(2) What is the conditional probability that a repair takes atleast 10 hours given that its duration exceeds 9 hours? (8)

12. (a) (i) The joint probability density function of the random variable (X, Y) is given by $f(x, y) = Kxye^{-(x^2+y^2)}$, $x > 0$, $y > 0$.

Find the value of K and Cov (X, Y). Are X and Y independent? (8)

(ii) If X and Y are uncorrelated random variables with variances 16 and 9. Find the correlation co-efficient between $X + Y$ and $X - Y$. (8)

Or

(b) (i) Let (X, Y) be a two dimensional random variable and the probability density function be given by

$$f(x, y) = x + y,\ 0 \le x,\ y \le 1$$

Find the p.d.f of $U = XY$. (8)

(ii) Let $X_1, X_2, \ldots X_n$ be Poisson variates with parameter $\lambda = 2$ and $S_n = X_1 + X_2 + \ldots X_n$ where $n = 75$. Find $P[120 \le S_n \le 160]$ using central limit theorem. (8)

13. (a) (i) If $\{X(t)\}$ is a WSS process with autocorrelation $R(\tau) = Ae^{-\alpha|\tau|}$, determine the second order moment of the $RV\{X(8) - X(5)\}$. (8)

(ii) If the WSS process $\{X(t)\}$ is given by $X(t) = 10\cos(100t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$, prove that $\{X(t)\}$ is correlation ergodic. (8)

Or

(b) (i) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is

(1) more than 1 minute

(2) between 1 minute and 2 minute and

(3) 4 min or less. (8)

(ii) Suppose that $X(t)$ is a Gaussian process with $\mu_x = 2$, $R_{xx}(\tau) = 5e^{-0.2|\tau|}$. Find the probability that $X(4) \le 1$. (8)

14. (a) (i) A stationary random process $X(t)$ with mean 2 has the auto correlation function $R_{XX}(\tau) = 4 + e^{-\frac{|\tau|}{10}}$. Find the mean and variance of $Y = \int_0^1 X(t)\,dt$. (8)

(ii) Find the power spectral density function whose autocorrelation function is given by $R_{XX}(\tau) = \frac{A^2}{2}\cos(\omega_0 \tau)$. (8)

Or

(b) (i) The cross-correlation function of two processes $X(t)$ and $Y(t)$ is given by $R_{XY}(t,t+\tau)=\frac{AB}{2}\{\sin(\omega_o\tau)+\cos\omega_o[(2t+\tau)]\}$ where A, B and ω_0 are constants. Find the cross-power spectrum $S_{XY}(\omega)$. (8)

(ii) Let $X(t)$ and $Y(t)$ be both zero-mean and WSS random processes Consider the random process $Z(t)$ defined by $Z(t)=X(t)+Y(t)$. Find

(1) The Auto correlation function and the power spectrum of $Z(t)$ if $X(t)$ and $Y(t)$ are jointly WSS.

(2) The power spectrum of $Z(t)$ if $X(t)$ and $Y(t)$ are orthogonal. (8)

15. (a) (i) Consider a system with transfer function $\frac{1}{1+j\omega}$. An input signal with autocorrelation function $m\delta(\tau)+m^2$ is fed as input to the system. Find the mean and mean-square value of the output. (8)

(ii) A stationary random process $X(t)$ having the autocorrelation function $R_{XX}(\tau)=A\delta(\tau)$ is applied to a linear system at time $t=0$ where $f(\tau)$ represent the impulse function. The linear system has the impulse response of $h(t)=e^{-bt}u(t)$ where $u(t)$ represents the unit step function. Find $R_{YY}(\tau)$. Also find the mean and variance of $Y(t)$. (8)

Or

(b) (i) If $\{X(t)\}$ is a WSS process and if $Y(t)=\int_{-\infty}^{\infty}h(\xi)X(t-\xi)d\xi$ then prove that

(1) $R_{XY}(\tau)=R_{XX}(\tau)*h(\tau)$ where $*$ stands for convolution.

(2) $S_{XY}(\omega)=S_{XX}(\omega)H^*(\omega)$. (8)

(ii) If $\{N(t)\}$ is a band limited white noise centered at a carrier frequency ω_0 such that $S_{NN}(\omega)=\begin{cases}\frac{N_0}{2}, & \text{for } |\omega-\omega_0|<\omega_B \\ 0, & \text{elsewhere.}\end{cases}$ Find the autocorrelation of $\{N(t)\}$. (8)
